Number System

Classification of Numbers :-

Complex No. $a + ib$ (a: Real, ib: Imaginary)

- **Real Number** → which can be denoted on number line
  - **Decimal**
  - **Integer**
- **Imaginary No.** → Can not be denoted on number line.

Decimal

- **Rational No.** → which can be written in $\frac{p}{q}$ form where $p, q \to$ Integers, $(q \neq 0)$

  Ex. $\frac{2}{3}$, $\frac{-4}{5}$, 0.444, $\frac{22}{7}$, $\sqrt{9}$, $\sqrt[3]{8}$
  - **Terminating Decimal**
    - $0.2 = \frac{1}{5}$
    - $0.67 = \frac{67}{100}$
  - **Non-Terminating Repeating decimal**
    - $0.444\ldots = 0.\bar{4} = \frac{4}{9}$
    - $0.525252\ldots = 0.\overline{56} = \frac{56}{99}$
- **Irrationdl No.** → Can not be written in $\frac{p}{q}$ form.

  Ex. $0.14325693\ldots$

  $\sqrt{2}, \sqrt{3}, \sqrt{5}$

  $\pi = 3.141592\ldots$

  $2+\sqrt{2}$, $2-\sqrt{2}$, $2\sqrt{2}$

  ⇓

  Non-terminating Non-repeating decimal

Integer ⇒ All integers are rational No. $\left(\frac{p}{q}\right)$ where $q = 1$

- **+ve Integer** $\{1, 2, 3, 4 \dots \infty\}$
- **0** ⇓ Neither +ve Nor -ve
- **-ve Integer** $\{-\infty, \dots -3, -2, -1\}$

Natural No.

- **Even** — Divided by 2 $\{2, 4, 6, 8 \dots, -2, -4, \}$ $\{0\}$ — **2k form**
- **Odd** — Not divided by 2 $\{-5, -3, -1, 1, 3, 5 \dots\}$ — **2k ± 1 form**

⊕ **Whole No** $\{0, 1, 2, 3, 4 \dots \infty\}$ [Non-Negative Integer]

Natural No.

- **Prime No.** — Only 2 factors (1 & itself) — 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
  - 2 → Even Prime No. Smallest prime No.
  - 3, 5, 7 → Only pair of Consecutive odd prime No.
- **1** — Neither Prime Nor Composite
- **Composite No.** — more than 2 factors — 6, 15, 12 (1, 2, 3, 6; 1, 2, 3, 4, 6, 12)
  - ⇒ 4 → Smallest Composite No.
  - 9 → Smallest odd Composite No.

		Prime No.	Sum
1 - 25	⟶	9	100
1 - 50	⟶	15	328
1 - 75	⟶	21	712
1 - 200	⟶	46	
1 - 1000	⟶	168	
1 - 100	⟶	25	1060

⇒ Each Prime number can be written in $(6k \pm 1)$ form But every $(6k \pm 1)$ form may not be necessarily Prime No.

Ex:- 13 → $6 \times 2 + 1$ (Prime)

25 → $6 \times 4 + 1$ (Not Prime)

⇒ **Co-prime Numbers / Relatively Prime** :- Two numbers in which nothing is common ie their HCF = 1

if HCF $(a, b) = 1$ ⇒ a, b are ~~Comp~~ Co-Prime No.

Ex. (4, 5), (2, 3), (5, 9), (11, 17)

⇒ **Twin-Prime numbers** :- Two Prime numbers with a difference of 2.

Ex. (3, 5), (5, 7), (11, 13)

⇒ **Perfect Numbers** :- If the sum of all factors (excluding that no.) is equal to that number.

Ex. 6, 28, 496, 8128

$6 \rightarrow 1, 2, 3, \overset{\times}{6} \Rightarrow 1+2+3 = 6$

$28 \rightarrow 1, 2, 4, 7, 14, \overset{\times}{28} \Rightarrow 1+2+4+7+14 = 28$

[4] Which of the following numbers is not a prime number?

(a) 197 (b) 313 (c) 439 (d) 391 ✓

$\div 17, 23$

[5] x, y and z are prime number and $x+y+z = 38$. What is the maximum value of x?

(a) 19 (b) 23 (c) 31 (d) 29

Soln:- Case I

$x+y+z = 38$

$2+3+z = 38$

$z = 33$

It is not a Prime No.

Case II

$x+y+z = 38$

$2+5+z = 38$

$z = \boxed{31}$ Ans

[6] The sum of a natural number and its square equals the Product of the first three prime numbers. The number is:

Soln:-

$n + n^2 = 2 \times 3 \times 5$

$n^2+n-30 = 0$

$(n+6)(n-5) = 0$

$n = 5, -6$ ✗

$\boxed{n=5}$ Ans

[7] The number which can be written in the form of $n(n+1)(n+2)$, where n is a natural number, is:

(a) 7 (b) 5 (c) 3 (d) 6 ✓

Soln:- $n=1 \Rightarrow 1 \times 2 \times 3 = \boxed{6}$

[8] A rational number between $\frac{3}{4}$ and $\frac{3}{8}$ is:

(a) $\frac{7}{9}$ ⇓ 0.77 (b) $\frac{7}{3}$ ⇓ 2.33 (c) ✓ $\frac{5}{9}$ ⇓ 0.55 (d) $1\frac{9}{16}$ ⇓ $\frac{25}{16} = 1.5$

$\frac{3}{4} = 0.75$ $\frac{3}{8} = 0.375$

between = 0.55 ✓

34 Which number is divisible by both 9 and 11 ?

(a) 10,089 (b) 10,098 (c) 10,108 (d) 10,087

Sum = 1+9+8 ÷ 9 ✓

(1+8) - 9 = 0 ÷ 11 ✓

35 If 8-digit number 4432A43B is divisible by 9 and 5, then the sum of A and B is equal to:

Soln:- 4432A43B → 0/5

A+B+2 = 9

A+B = 7

36 If the number 62783xy is divisible by both 8 and 5, then the smallest possible value of x and y is:

Soln:- 62783xy → 0/5

$\frac{3x0}{8}$, $\frac{3x5}{8}$ ✗ not Possible

Unit digit = 8 ✗

⇒ x = 2, 6

y = 0

37 What least value must be assigned to * so that the number 63576*2 is divisible by 8 ?

Soln: $\frac{6*2}{8}$ ⇒ * ⇒ **3** Ans

38 If 142N is divisible by 12, then what is the value of N ?

Soln:- 12 = 4 × 3

142N

7+N

$\frac{2N}{4}$ ⇒ **N = 8**

8, is possible for both 4, 3 divisible

Remainder

Dividend

Divisor → 7 ⟌ 67 (9 ← Quotient

-63

4 ← Remainder

Dividend = Divisor × Quotient + Remainder

$[D = Q \times d + r]$

⇒ Rem < Divisor [Rem. always less than divisor]

Ex. $\frac{19}{5}$ ⇒ Rem. 4

Ex. $\frac{206}{11}$, R = 8

Ex. $\frac{120}{15}$, R = 0

Effect of Simplify :-

$\frac{28}{12}$ ⇒ Rem. = 4

↓ Simplify by 4 *

$\frac{7}{3}$ ⇒ Rem. = 1 —(actual Rem. × 4)→ 4

(Simplify by 4)

⇒ Remainder of Powers :-

Ex:- $\frac{(8)^3}{7} \Rightarrow \frac{\overset{①}{8}\times\overset{①}{8}\times\overset{①}{8}}{7} \Rightarrow$ Rem. $\boxed{1}$

↳ m-II $\frac{\overset{①}{(8)}^3}{7} \Rightarrow (1)^3 = \boxed{1}$

Ex:- $\frac{\overset{②}{(9)}^9}{7} \Rightarrow \frac{(2)^9}{7} \Rightarrow \frac{(8)^3}{7} \Rightarrow (1)^3 = \overset{\text{Rem.}}{\boxed{1}}$

97 When $(77^{77} + 77)$ is divided by 78, the remainder is: [CHSL-2020]

Soln :- $\frac{\overset{(-1)}{77}^{77} + \overset{(-1)}{77}}{78} \Rightarrow (-1)^{77} + (-1) \Rightarrow \overset{-\text{ve Rem}}{\boxed{-2}}$

+ve $\Rightarrow 78 - 2 = \boxed{76}$ Ans

98 $(7^{19} + 2)$ is divided by 6, the remainder :

Soln :- $\frac{(1)^{19}}{7^{19} + 2} \Rightarrow 1 + 2 = \boxed{3}$ Ans
6

99 For any integral value of n $3^{2n} + 9n + 5$ when divided by 3 will leave the remainder.

Soln :- $\frac{3^{2n} + 9n + \overset{(2)}{5}}{3} \Rightarrow \boxed{2}$ Ans

100 If $29^{41} + 37^{41}$ is divided by 33, then the remainder is:

Soln:-

m-I $29^{41} + 37^{41}$ [$\because a^n + b^n$ is divisible by $(a+b)$ if $n \to$ odd]

$29+37 \Rightarrow \underline{66}$ is multiple of 33

So **Rem = 0**

m-II $\dfrac{29^{41} + 37^{41}}{33} \Rightarrow \dfrac{(-4)^{41} + (4)^{41}}{33} \Rightarrow \overset{a+b}{-4+4} = \boxed{0}$

101 What is the remainder when $(127^{97} + 97^{97})$ is divided by 32 ?

Soln:

$$\frac{127^{97} + 97^{97}}{32} \Rightarrow \frac{(127+97)}{32} = \frac{224}{32} \Rightarrow \boxed{\text{Rem} = 0}$$ Ans

102 What is the remainder when we divided $5^{70} + 7^{70}$ by 74

Soln $5^{70} + 7^{70} \to (5^2)^{35} + (7^2)^{35} \to 25^{35} + 49^{35}$

Divi. by $\to 25 + 49 = 74$

Rem = 0 Ans

103 What is the remainder when $(576^{273} - 412^{273})$ is divisible by 86 ?

Soln:- Div. by $\to 756 - 412 = 344 \to 4 \times 86$

$\therefore$ **Rem = 0**

⇒ If Power is odd

$a^n + b^n + c^n + d^n \ldots \longrightarrow$ is divisible by $(a+b+c+d)$

Ex:- $\frac{16^{11} + 17^{11} + 18^{11} + 19^{11}}{70} \Rightarrow (16+17+18+19) = 70$

$\therefore$ Rem = 0

104 Find the remainder: $\frac{1^{23} + 2^{23} + 3^{23} + \ldots + 70^{23}}{71}$

Soln:- Div. by $\rightarrow 1+2+3+4+\ldots 70 = \frac{70 \times 71}{2}$

$= \frac{70 \times 71}{2 \times 71}$ Rem = 0

Fermat's Theorem :→

$$\frac{a^{P-1}}{P} = 1 \text{ (Rem.)}$$

P = Prime no.

a, P → Coprime No.

Ex:- $\frac{40^{18}}{19} \rightarrow$ R = 1

Ex:- $\frac{90^{82}}{83} \rightarrow$ R = 1

Ex:- $\frac{68^{30}}{31} \rightarrow$ R = 1

Ex:- $\frac{93^{12}}{11} \rightarrow$ R = 1

Ex:- $\frac{2^{10}}{11} \rightarrow$ R = 1

Ex:- $\frac{2^{4}}{5} \rightarrow$ R = 1

$\Rightarrow \frac{82^{54}}{19} \rightarrow \frac{(82^{18})^3}{19} \rightarrow (1)^3 = \boxed{\text{Rem. } 1}$

$\Rightarrow \frac{2^{89}}{89} \rightarrow \frac{(2^{88}) \times 2^1}{89} \Rightarrow 1 \times 2 = \overset{\text{rem.}}{\boxed{2}}$

$\Rightarrow \frac{3^{99}}{97} \rightarrow \frac{\overset{①}{3^{96}} \times \overset{㉗}{3^3}}{97} \Rightarrow \overset{\text{rem.}}{\boxed{27}}$

$\Rightarrow \frac{5^{32}}{31} \rightarrow \frac{\overset{①}{5^{30}} \times \overset{㉕}{5^2}}{31} \Rightarrow \overset{\text{rem.}}{\boxed{25}}$

$\Rightarrow \frac{13^{44}}{7} \rightarrow \frac{\overset{①}{(13^6)^7} \times 13^2}{7} \Rightarrow \frac{\overset{-1}{(13)^2}}{7} = \overset{\text{Rem.}}{\boxed{1}}$

105 The remainder when 3^{21} is divided by 5 is:

Soln:- $\frac{3^{21}}{5} \rightarrow \frac{\overset{①}{(3^4)^5} \times 3}{5} \Rightarrow \boxed{3}$ Ans

106 Find the remainder: $\frac{9^{111}}{13}$

Soln:- $\frac{9^{111}}{13} \rightarrow \frac{(9^{12})^9 \times 9^3}{13} \Rightarrow \frac{729}{13} \Rightarrow \boxed{R=1}$

107 $\frac{4^{2007}}{17} \rightarrow \frac{\overset{①}{(4^{16})^{125}} \times 4^7}{17} \Rightarrow 4^7 \Rightarrow \frac{\overset{-1}{16} \times \overset{-1}{16} \times \overset{-1}{16} \times \overset{④}{4}}{17}$

$\Rightarrow -4 / 17-4 = \overset{\text{Rem.}}{\boxed{13}}$ Ans